

Breakup probabilities and the optical potential in elastic deuteron scattering

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Abstract

We investigate the effect of the deuteron breakup on the optical potential for the elastic scattering of 56 MeV deuterons from ^{51}V . The breakup probabilities calculated within the post-form distorted wave Born-approximation theory of breakup reactions are fitted to derive

the contribution to the optical potential from the breakup channels. The breakup potentials are found to have large real as well as imaginary parts. Thus the dynamical polarization potential due to the breakup process is expected to modify strongly the real part of the optical potential calculated by the double folding model.

1 Introduction

The folding model with realistic effective nucleon-nucleon interactions has given a good insight into the nucleus-nucleus interaction (see e.g. a recent review by Brandan and Satchler [1]). However, when a light ion interacts with other ions there is also a possibility of the breakup of the projectile into two or more fragments. If the breakup channel is strong, it will affect not only the imaginary potential but also the real one. This leads to a dynamical polarisation potential (DPP) which has to be added to the real potential obtained by the folding model. The DPP due to the breakup process can, for example, be estimated from the adiabatic model proposed by Johnson and Soper [2], or by more sophisticated continuum discretised coupled channels techniques [3]. The DPP is required to describe the elastic scattering of the weakly bound projectiles like ${}^6\text{Li}$ and ${}^9\text{Be}$ in the folding model with reasonable renormalisation factors [1].

Experimental studies have shown that the breakup probabilities increase drastically with energy even for tightly bound projectiles [4, 5]. For example, the cross section for the breakup of the α particle increases by, at least, an order of magnitude as the beam energy is varied from 65 MeV to 140 MeV [6, 7]. At higher beam energies (≥ 140 MeV) the breakup cross section can be as large as 25% of the total reaction cross section. Thus the DPP could be required also for tightly bound projectiles for beam energies above 30 MeV/A.

In a recent report [8] one of us investigated the contributions to the α -particle optical potential from the $(\alpha, {}^3\text{He})$ breakup reaction on ${}^{62}\text{Ni}$ target at the incident energy of 172.5 MeV. The calculations proceed in two steps. First the breakup probabilities are calculated within a theory which is formulated in the frame-work of the post form distorted-wave Born-approximation (PFDWBA). This theory has been found to reproduce the experimental data on the breakup of light projectiles extremely well [9]. In the second step, these probabilities are fitted to generate the breakup part of the optical potential which is assumed to consist of two parts; one which is entirely due to the breakup of the projectile and the another independent of it. It was found in [8] that breakup contributes substantially to the optical potential. However, in [8], the DPP for the alpha elastic scattering was obtained by fitting only to the $(\alpha, {}^3\text{He})$ breakup channel. For a complete determination of the DPP due to the breakup process the breakup probabilities for all the α -breakup channels, namely, (α, p) , (α, n) , (α, d) , $(\alpha, {}^3\text{He})$ and (α, t) should be calculated and fitted.

Before taking up this rather ambitious task, which nobody has investigated so far, we considered it worthwhile to perform calculations for the simple system of the deuteron to test the method in detail. In this case there are only two breakup channels, (d, p) and (d, n) and the experimental data on the deuteron breakup (see e.g. [10]) is quite comprehensive. In this paper, we investigate the scattering of 56 MeV deuterons from ${}^{51}\text{V}$. The breakup prob-

abilities for the (d, p) and (d, n) channels are calculated for this particular case. These are fitted to determine the DPP due to the breakup process.

In sections 2 we describe the calculation of the breakup probabilities in the optical model. The calculation of the same within the PFDWBA theory of breakup is presented in section 3. The results for the DPP obtained by fitting the total and elastic breakup probabilities are discussed in sections 4. Our conclusions are presented in section 5.

2 Calculations of breakup probabilities from the optical model.

The calculations are based on the same principles as in Ref. [8]. We assume that the optical potential $V(r)$ is known from the phenomenological analyses of elastic scattering data and solve the radial Schrödinger equation

$$\frac{d^2 y_\ell(r)}{dr^2} + [k^2 - U(r) - \frac{\ell(\ell+1)}{r^2}] y_\ell(r) = 0, \quad (1)$$

where k is the wavenumber and $U(r)$ is related to the optical potential $V(r)$ through $U(r) = (2m/\hbar^2)V(r)$. The solutions for the radial wavefunctions, $y_\ell(r)$, are normalised according to

$$y_\ell(r) = e^{i\delta_\ell} [\cos \delta_\ell F_\ell(kr) + \sin \delta_\ell G_\ell(kr)], \quad (2)$$

where F_ℓ and G_ℓ are the regular and irregular Coulomb functions. With this normalisation the partial wave amplitudes, T_ℓ , are written in terms of the

phase-shifts δ_ℓ as

$$T_\ell = e^{i\delta_\ell} \sin \delta_\ell \quad (3)$$

The partial wave amplitudes may also be calculated from the relation

$$T_\ell = -k^{-1} \int_0^\infty F_\ell(kr) U(r) y_\ell(r) dr \quad (4)$$

Now we define the optical potential due to breakup as $U_{bu}(r)$ and write the full potential $U(r)$ as $(U(r) - U_{bu}(r)) + U_{bu}(r)$. With this definition, Eq. 1 can be recast as

$$\frac{d^2 y_\ell(r)}{dr^2} + [k^2 - (U(r) - U_{bu}(r)) - U_{bu}(r) - \frac{\ell(\ell+1)}{r^2}] y_\ell(r) = 0 \quad (5)$$

A discussion of this problem may be found in Ref. [11, 12].

By solving the Schrödinger equation for the "bare" potential $U(r) - U_{bu}(r)$,

$$\frac{d^2 v_\ell(r)}{dr^2} + [k^2 - (U(r) - U_{bu}) - \frac{\ell(\ell+1)}{r^2}] v_\ell(r) = 0, \quad (6)$$

we obtain the radial wavefunctions $v_\ell(r)$, the phase shifts δ'_ℓ , and the partial wave amplitudes, T'_ℓ .

The relation between the partial wave amplitudes are

$$T_\ell = T'_\ell + T_\ell^{bu} \quad (7)$$

The partial wave amplitudes for breakup are thus given by the difference between the partial wave amplitudes obtained for the potentials $U(r)$ and $U(r) - U_{bu}(r)$, respectively. These can also be calculated from the formula (see e.g. [11, 12])

$$T_\ell^{bu} = -k^{-1} \int_0^\infty v_\ell(kr) U_{bu}(r) y_\ell(r) dr \quad (8)$$

It should be noted that the integral in Eq. 8 contains the radial wavefunctions v_ℓ , obtained with the bare potential $U(r) - U_{bu}(r)$, as well as the radial wavefunctions y_ℓ , obtained with the full potential $U(r)$. The assumption implicate therein is that the bare potential cannot lead to the breakup reaction.

The breakup probability for a certain ℓ -value is determined from the partial wave amplitudes T_ℓ^{bu} according to

$$\text{Breakup probability} = |T_\ell^{bu}|^2 \quad (9)$$

These breakup probabilities are compared with those calculated within the PFDWBA theory of breakup reactions (discussed in the next section) to determine the breakup potential, U_{bu} . In our procedure, this potential (with certain *a priori* assumed form) is varied to reproduce the breakup probabilities calculated within PFDWBA. It may be remarked here that when the potential $U_{bu}(r)$ is varied, new values of T'_ℓ have to be calculated from the solution of eq. 6, whereas the values of T_ℓ remain unchanged.

It is interesting to note that the reaction cross section in the elastic scattering is known from the solutions of eq. (1). Furthermore the total breakup cross section can be calculated from the breakup probabilities. Even then, the reaction cross section for the bare potential $U(r) - U_{bu}(r)$, will depend on the shape of the potential, $U_{bu}(r)$.

In Ref. [8], the peripheral dominance of the breakup process made us parametrize the breakup potential as a Gaussian with the addition of a Woods-Saxon form factor. In this work, however, this preconceived view is avoided and a Fourier-Bessel (FB) expansion

$$U_{bu}(r) = \sum_{n=1}^N a_n \frac{\sin(n\pi r)}{R}, \quad (10)$$

is used for both the real as well as imaginary parts of the breakup potential. The radius R was kept fixed at 18 fm in all calculations.

3 Microscopic calculations of breakup probabilities.

In the theory of the breakup reaction ($d \rightarrow p + n$) formulated within the framework of the post form distorted-wave Born-approximation, the probability of breakup $P_{\ell_a}^{b-up(d,p)}$ is defined by [13]

$$\begin{aligned} \sigma_{total}^{b-up}(d, p) &= \int d\Omega_p dE_p \frac{d^2\sigma(d, p)}{d\Omega_p dE_p} \\ &= \frac{\pi}{k_d^2} \sum_{\ell_d} (2\ell_d + 1) P_{\ell_d}^{b-up(d,p)}, \end{aligned} \quad (11)$$

where $\frac{d^2\sigma(d,p)}{d\Omega_p dE_p}$ is the double differential cross section for the inclusive breakup reaction (d,p), which is the sum of the elastic and inelastic breakup modes. These are given by

$$\frac{d^2\sigma(elastic)}{d\Omega_p dE_p} = \frac{\mu_d \mu_p \mu_n}{(2\pi)^5 \hbar^6} \frac{k_p k_n}{k_d} \sum_{\ell_n m_n} |T_{\ell_n m_n}|^2, \quad (12)$$

where the T-matrix $T_{\ell_n m_n}$ is

$$T_{\ell_n m_n} = D_0 \int d^3 r \chi_p^{(-)*}(\mathbf{k}_p, \frac{A}{A+1} \mathbf{r}) \frac{\chi_{\ell_n}}{k_n r} Y_{\ell_n m_n}(\hat{r}) \chi_d^{(+)}(\mathbf{k}_d, \mathbf{r}) \Lambda(r) P(r) \quad (13)$$

D_0 is the well known zero range constant for the $d \rightarrow p + n$ vertex. $\Lambda(r)$ and $P(r)$ are the finite range and nonlocality correction factors respectively. χ^\pm are the optical model wave functions in the respective channels with \mathbf{k} 's being the corresponding wave vectors.

The inelastic breakup cross section is given by

$$\frac{d^2 \sigma(inelastic)}{d\Omega_p dE_p} = \frac{\mu_d \mu_p \mu_n}{(2\pi)^5 \hbar^6} \frac{k_p k_n}{k_d} \sum_{\ell_n m_n} (\sigma_{\ell_n}^{reaction} / \sigma_{\ell_n}^{elastic}) | T_{\ell_n m_n} - T_{\ell_n m_n}^0 |^2 \quad (14)$$

In this equation $\sigma_{\ell_n}^{reaction}$ and $\sigma_{\ell_n}^{elastic}$ are the reaction and elastic scattering cross sections for the neutron - target system corresponding to the partial wave ℓ_n respectively. The T matrix $T_{\ell_n m_n}^0$ is defined in the same way as Eq. (13) with the elastics scattering wave function χ_{ℓ_n} being replaced by the spherical Bessel function. It may be noted that Eq. (14) includes contributions from all the inelastic channels of the neutron + target system (see e.g [9] for complete detail).

The angular integration in Eq. (13) is performed by introducing the partial wave expansion of the distorted waves and using the orthogonality of the spherical harmonics. The resulting slowly converging radial integrals are evaluated very effectively by following the contour integration technique of Vincent and Fortune [16].

We require the optical potentials in the incident and outgoing channels as

input in our calculations. In the results presented in this paper, the deuteron optical potentials were taken from the global sets given by Daehnick, Childs and Vrcelj [14] whereas the potentials of Becchetti and Greenlees [15] were used in the neutron and proton channels.

The total breakup probability P_{ℓ_d} is defined as following:

$$P_{\ell_d}^{b-up,d} = P_{\ell_d}^{b-up(d,pn)}(elastic) + P_{\ell_d}^{b-up(d,p)}(inelastic) + P_{\ell_d}^{b-up(d,n)}(inelastic) \quad (15)$$

In Fig. 1 we show the results for the breakup probability for the deuteron incident on a ^{51}V target at the beam energy of 56 MeV. We can see that the (d,p) and (d,n) breakup probabilities are similar in shape and absolute magnitude. The elastic breakup probability is much smaller and shows a different behaviour as a function of ℓ_d .

As discussed in Ref. [9] the total cross section can also be expressed as

$$\sigma_d^{breakup} = 2\pi \int db \, b \, P_{\ell_d}^{b-up,d} \quad (16)$$

where the impact parameter, b , is related to the angular momentum, ℓ_d , and the wavenumber k_d , through $b = (\ell_d + 1/2)/k_d$.

The cross sections for the inelastic (d,p), inelastic (d,n) and elastic (d,pn) breakup processes were found to be 290 mb, 214 mb, and 122 mb respectively. This leads to a total breakup cross section of 627 mb. It may be noted that our total (d,p) breakup cross section (which is the sum of inelastic (d,p) and

elastic (d,pn) cross sections) is 412 mb which is in reasonable agreement with the measured value of 481 mb reported by Matsuoka et al. [10].

4 Results and discussions

The fitting procedure was the same way as that described in Ref. [8]. However, the imaginary part of the breakup potential, U_{bu} , was assumed to be less than that of the full potential $U(r)$, so that $(U(r) - U_{bu}(r))$ was absorptive for all the radii. The errors associated with the calculated breakup probabilities in the optical model (Eq. (9)) were about 10%. In the fitting procedure, 14 coefficients (a^n as defined in Eq. 10) were varied simultaneously for each potential to get a minimum in the χ^2 .

In the calculations for (α - ^3He) breakup [8] it was found that the gross properties of the breakup probabilities could be reproduced with a purely real as well as a purely imaginary breakup potential. An imaginary part of the potential was needed for fitting them at only small ℓ -values. However, in case of the deuteron we found it impossible to reproduce the breakup probabilities (P_{ℓ_d}) without a complex breakup potential. One of the reasons for this could be the fact that the breakup probabilities in this case are considerably larger than those for the (α - ^3He) reaction. Fig. 5 shows the best fit obtained with a purely real breakup potential whereas Fig. 3 with a purely imaginary one. As can be seen from these figures, the imaginary potential is very important for reproducing the breakup probabilities at small

ℓ -values, while the real part is required to reproduce these for the grazing partial waves and beyond. This supports the observation made in [17] that the real potential, due to non eikonal effects, increases the absorption at large radii and enhances strongly the peripheral collisions.

The best fit result obtained with a complex potential is shown in Fig. 4. Both potentials include 14 terms in the FB expansion. It can be seen that the breakup probabilities calculated within PFDWBA (shown in Fig. 1), are reproduced very well. The imaginary part of the breakup potential is found to be very strong. This is not surprising since the inelastic interactions (which could cause the breakup of the projectile) are taken into account by the imaginary part of $U(r)$. These will automatically be included in the imaginary part of U_{bu} . In the folding model calculations of the elastic scattering where the imaginary part of the potential is phenomenologically determined, it is mainly the real potential which is of interest. Therefore, our result suggest that the inclusion of the potential, U_{bu} , in these calculations will lead to a strong modification to the real part of the folding model potentials.

Yabana et al [18] have investigated the effects of breakup for the elastic scattering of 80 MeV deuterons from ^{58}Ni . Both real and imaginary parts of the DPP obtained by these authors are weaker than those obtained in our work. It may be remarked here that, these authors have neglected the Coulomb breakup process, which can be quite large for the deuteron target interaction [20, 9]. In our work, on the other hand, this is included on the

same footing as the nuclear breakup. Moreover, our breakup probabilities lead to the cross sections which agree well with the experimental data on the deuteron breakup.

We should, however, stress that, there are (as in optical model calculations) ambiguities in the breakup potentials obtained by us. For instance, in Fig. 5 we show the results obtained when the number of terms in the real potential is decreased from 14 to 4. The fit to the PFDWBA breakup probabilities is still reasonable. This therefore, makes it difficult to arrive at a definite conclusion about the shapes of the DPP. Nevertheless, it is interesting to note that the real part of the DPP obtained in this way is still larger than those of Yabana et al. [18]

Next we discuss the breakup probability for the elastic breakup (a process in which the target nucleus remains in the ground state). In theories like the one suggested by Bertsch, Brown and Sagawa [19] the reaction cross section in nucleus-nucleus collisions are calculated from collisions between nucleon-nucleon pairs in the projectile and target nuclei. In such approaches it is necessary to include a correction due to the elastic breakup of the projectile. We term the corresponding potential as the "dissociation" potential (U_{dis}) to separate it from the potential U_{bu} defined earlier. It would also be worthwhile to study how the dissociation affects the optical potential and the reaction cross section. U_{dis} is obtained by fitting (in the same way as described above) the elastic breakup probabilities as calculated by PFDWBA theory (see Fig.

1). We stress that these include both Coulomb and nuclear breakup as well as their interference terms.

The results obtained with a purely real U_{dis} are shown in Fig. 6. The fits to the elastic breakup probabilities are satisfactory for the grazing partial waves. However, those for the lower partial waves are poorer. On the other hand attempts to fit them with a purely imaginary U_{dis} resulted in a very bad agreement. Therefore we reduced the real potential shown in Fig. 6 and tried to reproduce the data by a variation of the imaginary potential. The results from this search is shown in Fig. 7 and it is evident that the maximum is very badly reproduced. However, when both the real and imaginary potentials are varied a good fit to the elastic breakup probabilities are obtained, even if the shape of the imaginary part of U_{br} so obtained looks somewhat unusual. Figure 8 shows one of the best fit potentials obtained in this way.

As discussed in section 2, the partial wave amplitude for the potential $U(r)$ is given by the sum of the partial wave amplitudes the potentials, $U(r) - U_{bu}(r)$ and $U_{bu}(r)$. This is not true for the reaction cross sections calculated from each potential separately, since these are sensitive to interference effects. The reaction cross section for the full potential is 1571 mb. With the real potential shown in Fig. 6 the reaction cross section for the potential, $U(r) - U_{bu}(r)$, is found to be 1405 mb. The difference (166 mb) is somewhat larger than the value (122 mb) obtained for the total elastic breakup cross section

using Eq. (11). With the complex potential shown in Fig. 8, the reaction cross section without breakup potential does not decrease, instead it goes to a value of 2173 mb. These examples show the importance of including the effects of the dissociation in the optical model. The interference effects are important and have to be treated correctly.

Our calculations, therefore, indicate that the dissociation of the deuteron should give a substantial contribution to the dynamical polarisation potential. The assumption that the optical potential, obtained from a fit to elastic scattering data, include effects of dissociation and inelastic breakup in a correct way must be questioned. There are examples, as in Refs. [13], where the absorption due to breakup is even larger than the total absorption predicted by the optical model for large ℓ -values. Since the elastic breakup is considerably larger than the inelastic breakup for large ℓ -values, this indicates that the optical models should be modified with a contribution of the dissociation process, which may not necessarily have an effect on the angular distributions for elastic scattering.

5 Conclusion

In conclusion, our study of the effects of the breakup on the optical potential for the elastic scattering of 56 MeV deuterons from ^{51}V shows that breakup gives a substantial contribution to this potential. We found that the strong enhancement of the peripheral collisions requires that this contribution has

a real part. Thus the folding model calculations should include a dynamical polarisation potential.

The investigations of the effects of the dissassociation also indicate a contribution to the dynamical polarisation potential. However, the conventional shapes used for the optical potential do not account for the dissassociation process in a correct way. Therefore the investigation should be repeated with more sophisticated optical model potentials with different shapes.

We plan to calculate breakup probabilities for all breakup channels for α -particles. We believe that these together will give total breakup probabilities of the same order as those of the deuterons studied here, at least for energies above 40 MeV/A. We see no reason why the breakup potential for α -particles should be purely imaginary when it is complex for deuterons. Therefore we believe that all light particles require dynamical polarisation potentials in folding model calculations at higher energies and that the energy dependence of effective interactions, which presently reproduce α -particle scattering, should to be modified.

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Figure 1: Calculated breakup probabilities in the scattering of 56 MeV deuterons from ^{51}V .

Figure 2: Results obtained with a real breakup potential. In the upper part, (a), the dashed curve shows the nominal potential, the solid curve the breakup potential and the dotted curve the difference. In the lower part, (b), the open circles show the total breakup probabilities from Fig.1 and the solid circles the fitted values

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Figure 3: Results obtained with an imaginary breakup potential. In the upper part, (a), the dashed curve shows the nominal potential, the solid curve the breakup potential and the dotted curve the difference. In the lower part, (b), the open circles show the total breakup probabilities from Fig.1 and the solid circles the fitted values

Figure 4: Results obtained with a complex breakup potential. In the upper parts, (a) and (b), the dashed curve shows the nominal potential, the solid curve the breakup potential and the dotted curve the difference. In the lower part, (c), the open circles show the total breakup probabilities from Fig.1 and the solid circles the fitted values

Figure 5: Results obtained when the number of terms in the FB-expansion of the real potential has been decreased from 14 to 4. The notations are the same as in Figure 4.

Figure 6: Results obtained with a real dissociation potential.

Figure 7: Results obtained with an imaginary dissociation potential. The real potential is in this case 50 % of the real potential shown in Figure 6

Figure 8: Results obtained with a complex dissociation potential.